m + n + p = ?

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The expression $\left(1+\sqrt[6]{26+15\sqrt{3}}-\sqrt[6]{26-15\sqrt{3}}\right)^6 = m+n\sqrt{p}$,

where $m, n,$ and p are positive integers and p isn't divisible by square of any prime.

Find $m+n+p$.

Solution by Arkady Alt, **San Jose**, **California**, **USA**.

Noting that $\left(2\pm\sqrt{3}\right)^3 = 26\pm 15\sqrt{3}$ and $2\pm\sqrt{3} = \dfrac{\left(\sqrt{3}\pm 1\right)^2}{2}$ we obtain

$$\sqrt[6]{26\pm 15\sqrt{3}} = \sqrt{\frac{\left(\sqrt{3}\pm 1\right)^2}{2}} = \frac{\sqrt{3}\pm 1}{\sqrt{2}}.$$

Hence, $\left(1+\sqrt[6]{26+15\sqrt{3}}-\sqrt[6]{26-15\sqrt{3}}\right)^6 = \left(1+\dfrac{\sqrt{3}+1}{\sqrt{2}}-\dfrac{\sqrt{3}-1}{\sqrt{2}}\right)^6 =$

$\left(\sqrt{2}+1\right)^6 = \left(\left(\sqrt{2}+1\right)^3\right)^2 = \left(5\sqrt{2}+7\right)^2 = 99+70\sqrt{2}$ and, therefore,

$m+n+p = 99+70+2 = 171.$